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MARS: improving risk-based portfolios using range-based volatility forecasts

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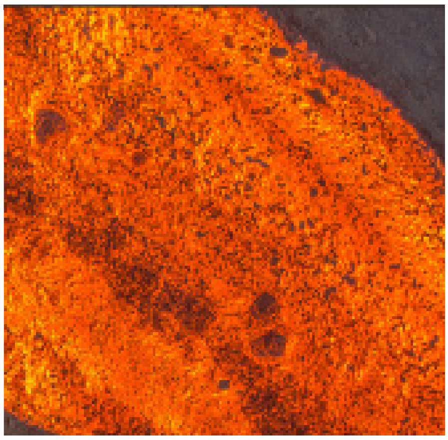

Leveraging intraday information to forecast volatility can help harvest additional performance when building multi-asset risk-based portfolios.

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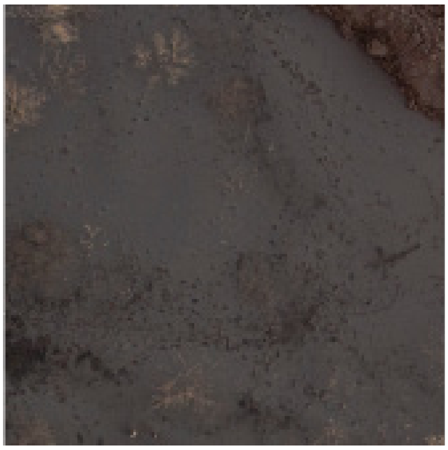

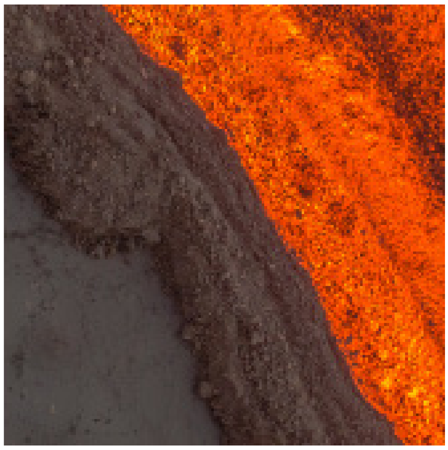

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MARS, our **M**ulti-**A**sset **R**esearch **S**eries that brings complex research back to Earth.

Introduction

In the previous editions of our Multi-Asset Research Series (MARS), we presented empirical facts advocating for the use of range-based measures to model spot volatility[1] and generate volatility forecasts[2]. In both instances, we found that volatility measures using intraday information were able to outperform standard volatility measures computed on close-to-close returns. In this edition, we conclude our range-based volatility triptych with an investment question: do these statistical advantages improve the return profile of risk-based portfolios?

As systematic risk-based investors, volatility is at the core of our investment process. However, the benefit of using range-based volatility measures in a risk-based portfolio construction is not obvious. It can seem intuitive – a better risk measure should improve the performance of a risk-based strategy – but there is no existing theory or straightforward empirical results leading to that conclusion.

In this paper, we investigate how volatility forecast improvements can help the performance of risk-based portfolios. The first section introduces how to leverage range-based volatility forecasts when computing Equal Risk Contributions portfolios. The second section considers the outperformance of portfolios built on range-based volatilities, emphasising that the benefit of using volatility models that account for intraday patterns is meaningful rather than being merely a statistical artefact. The third section evaluates this outperformance from the point of view of a risk-averse investor. The final section concludes and summarises the findings of our tri-volume focus on range-based volatility.

1 Chareyron, F., & Royer, J. (2023). A primer on range-based volatility estimators. *Lombard Odier Investment Managers - Multi Asset Research Series.*

2 Chareyron, F., Grignani, G., & Royer, J. (2023). Do volatility forecasts benefit from range-based measures?. *Lombard Odier Investment Managers - Multi Asset Research Series.*

Risk-based portfolio construction with range-based measures

When comparing the properties of competing volatility models, a natural exercise is to generate volatility forecasts and assess the relevance of the models by computing a loss function between the forecast series and a volatility proxy, as proposed by Patton (2011), and evaluating the significance of the difference between two models using Diebold and Mariano's procedure (1995). In this exercise, volatility models augmented by intraday data are often found to be superior.

For example, in a previous issue of MARS, we found that a GARCH model incorporating range-based measures (denoted GARCH-RB) was better at forecasting volatility than the standard conditional volatility model using a sample of S&P500 returns. However, it is not obvious that this statistical dominance translates into outperformance by portfolios constructed using this better model. Indeed, risk-based allocations are the result of sophisticated transformations of the covariance matrix that could temper the positive effect of including range-based measures. This important question has attracted far less attention. Focusing on Mean-Variance portfolios, Fleming et al. (2003), however, show that using intraday data can yield substantial improvements in performance, while De Nard et al. (2022) find similar results when using range-based measures.

More recently, new risk-based portfolios have been introduced. Amongst them, the Equal Risk Contribution (ERC) portfolio has been particularly successful in the financial industry and is indeed at the core of our investment process. In this section, we address how to leverage the GARCH-RB framework to build ERC portfolios. In line with existing literature (see Maillard et al. (2010)), we focus on volatility as the risk measure underlying our portfolio optimisation problem. The construction of the ERC amounts to finding a vector of weights $\boldsymbol{\omega}^* = \{\omega_1^*, \dots, \omega_n^*\}$ such that the contributions of each asset $i = 1, \dots, n$ to the total portfolio volatility are equal.

Let us denote $\sigma(\boldsymbol{\omega}) = \sqrt{\boldsymbol{\omega}'\Sigma\boldsymbol{\omega}}$ the volatility of the portfolio with weights $\boldsymbol{\omega}$, where Σ denotes the covariance matrix of asset returns. The volatility contribution of asset i is given by

$$c_i(\boldsymbol{\omega}) = \omega_i \frac{\partial \sigma(\boldsymbol{\omega})}{\partial \omega_i} = \omega_i \frac{\omega_i \sigma_i^2 + \sum_{j \neq i} \omega_j \rho_{ij} \sigma_i \sigma_j}{\sigma(\boldsymbol{\omega})}$$

where ρ_{ij} denotes the correlation between the returns of assets i and j and σ_i denotes the individual volatility of asset i. It follows that $\sum_{i=1,\dots,n} c_i(\boldsymbol{\omega}) = \sigma(\boldsymbol{\omega})$. Constructing the ERC portfolio thus amounts to finding $\boldsymbol{\omega}^*$ such that $c_i(\boldsymbol{\omega}^*) = c_j(\boldsymbol{\omega}^*)$ for all $i, j = 1, \dots, n$, this optimisation problem can be solved by sequential quadratic programming.

Of course, to obtain dynamic portfolios, one must replace the static covariance matrix Σ with its conditional counterpart Σ_t. In this setting, multivariate conditional volatility models such as the Conditional Correlation GARCH model of Engle (2002) allow us to easily obtain dynamic covariance matrices by specifying the individual volatility processes of each asset and decomposing the conditional covariance matrix as follows:

$$\Sigma_t = \begin{bmatrix} \sigma_{1,t} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \sigma_{n,t} \end{bmatrix} \mathrm{P}_t \begin{bmatrix} \sigma_{1,t} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \sigma_{n,t} \end{bmatrix}$$

where P_t is the conditional correlation matrix. Building on our previous MARS findings, we propose leveraging range-based volatility measures by modeling the individual volatility processes through a GARCH-RB specification[3]

$$\sigma_{i,t}^2 = \varpi_i + \alpha_i \varepsilon_{i,t-1}^2 + \beta_i \sigma_{i,t-1}^2 + \gamma_i \mathrm{RB}_{i,t-1}$$

for all $i = 1, \dots, n$. The benefits of intraday data can thus be assessed by comparing the performances of risk-based portfolios built using a GARCH-RB specification with portfolios constructed on a simple GARCH model.

[3] GARCH-RB specification is a particular GARCH-X model where the exogenous variable is the range-based volatility estimator of Garman-Klass (1980), see for example Francq and Thieu (2019).

Range-based measures can help build better portfolios

We evaluate the benefits of range-based measures by computing two ERC portfolios, one using a GARCH-RB specification and a competitor using only close-to-close data with a GARCH specification, on five assets spanning three asset classes. We use futures on major US (CME E-mini S&P 500 futures) and European (Eurex Euro Stoxx 50 futures) equity indices to build the equity exposure, while we use US (CBOT 10-Year US Treasury Note futures) and German (Eurex Euro - Bund futures) bond futures for the sovereign bond exposure. Additionally, we use crude oil futures (NYME Crude Oil WTI futures) for our commodities exposure.
We use daily data from June 1998 to June 2023 and retain a 5-year sample to estimate the initial necessary metrics to generate our portfolios. We then re-fit the models every day using an expanding window. As we want to focus our analysis on the links between volatility models and risk-based portfolios, we use a constant conditional correlation matrix and set it to the long-term empirical correlation matrix.

Arguably, portfolio managers should be more concerned by the forthcoming volatility over the portfolio holding period than by the spot volatility. We thus also consider two additional portfolios constructed using the same volatility models but based on forecasts of the covariance matrix over five days

$$\Sigma_{t:t+5|t} = \begin{bmatrix} \sigma_{1,t:t+5|t} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \sigma_{n,t:t+5|t} \end{bmatrix} \bar{\mathrm{P}} \begin{bmatrix} \sigma_{1,t:t+5|t} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \sigma_{n,t:t+5|t} \end{bmatrix}$$

where $\bar{\mathrm{P}}$ denotes the empirical correlation matrix and $\sigma_{i,t:t+5|t}$ denotes the individual volatility forecasts for assets $i = 1, \ldots, n$.

Table 1 presents performance metrics for the four portfolios.
We observe that, using either spot or forecasted covariance matrices, ERC portfolios including range-based volatility measures produce a higher Sharpe ratio than their standard GARCH concurrent. Additionally, the effect of the exogenous variable is stronger for the portfolios based on forecasts, consistent with the poor performance of standard GARCH models for generating forecasts of realised volatilities over longer horizons, as shown in our previous MARS research. Finally, in addition to being slightly more profitable, portfolios based on forecasts are remarkably more stable than portfolios built on spot. This can be explained by the ability of forecasts over five days to mitigate non-persistent spikes in volatilities.

TAB. 1 PERFORMANCE METRICS: STANDARD VS RANGE-BASED MODELS

	Spot		**Forecast**	
	GARCH	GARCH-RB	GARCH	GARCH-RB
Ann. return	3.20%	3.15%	3.14%	3.30%
Ann. vol	4.72%	4.62%	4.52%	4.67%
Sharpe Ratio	0.775	0.779	0.795	0.809
Max drawdown	-14.00%	-14.53%	-13.79%	-13.69%
Ann. turnover	6.91	8.29	2.38	2.63

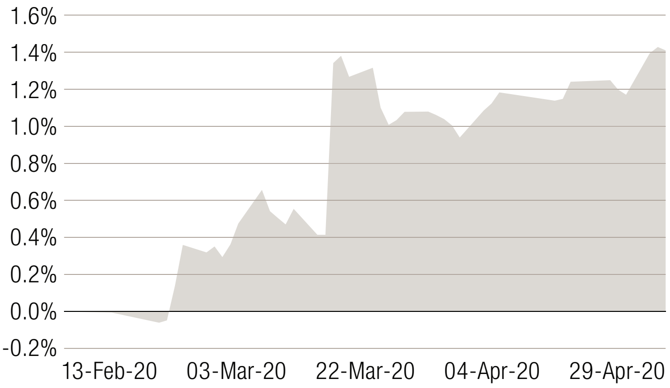
Source: LOIM. For illustrative purposes only. Ann. refers to annualised.

To illustrate the advantages of including range-based volatility measures, we focus on the Covid-19 market crash and recovery from February 2020 to May 2020. The left-hand chart of Figure 1 presents the outperformance of the portfolio incorporating intraday information benchmarked against the portfolio built using only close-to-close return. It shows that, during this period of high volatility, the former consistently outperforms the latter.

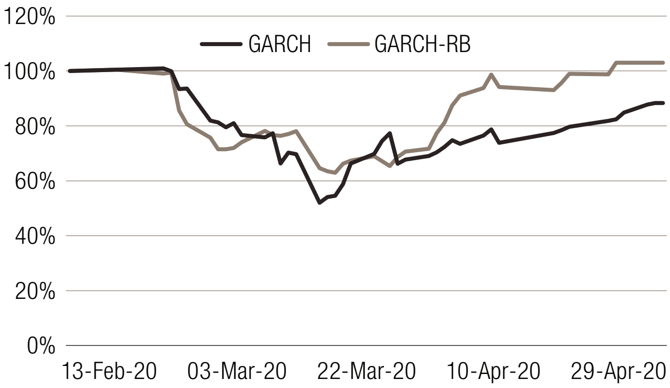
The right-hand chart of figure 1 shows the evolution of the equity allocation composed of S&P 500 and Euro Stoxx 50 futures of the two portfolios and rebased to a level in mid-February. One can note that including range-based measures in the covariance model allows us to diminish the risky-asset allocation at the beginning of the crisis, while the faster mean-reversion of the range-based measures help re-risk the portfolio and better capture the equity rally.

FIG. 1 PORTFOLIO COMPARISON DURING COVID-19

OUPERFORMANCE OF THE GARCH-RB VERSUS GARCH PORTFOLIO



EVOLUTION OF THE EQUITY ALLOCATION



Source: LOIM. For illustrative purposes only.

Assessing the outperformance from an investor perspective

Although risk-based portfolios incorporating intraday information appear to deliver a better risk-adjusted performance compared to portfolios constructed with standard volatility models, whether this outperformance is actually meaningful for a risk-averse investor remains uncertain. For example, if the outperformance occurs at a cost of incurring greater risks, even if the performance-to-risk ratio improves, an investor unwilling to bear more risk might not be interested in the outperformance embedded in the range-based portfolio construction. Additionally, even for similar risk levels, an investor may choose to sacrifice the additional performance linked to range-based measure if the risk materialises in a period that is more subject to risk aversion, for example a recession.

To investigate how range-based measures are appealing to rational investors, we propose to follow Fleming et al. (2001) and study the utility gains generated by choosing the GARCH-RB portfolio over the standard GARCH portfolio. Consider an investor with quadratic utility[4] investing at each date a fixed amount of wealth W_0 in a risk based portfolio φ. We can write the realised daily utility of this portfolio as

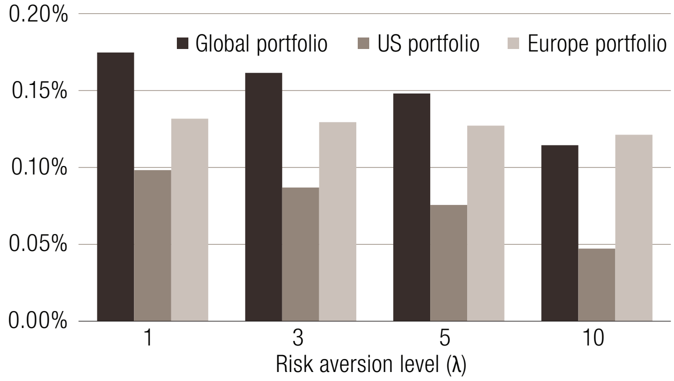
$$U(R_{\varphi,t}) = W_0\left((1+R_{\varphi,t}) - \frac{\lambda}{2(1+\lambda)}(1+R_{\varphi,t})^2\right)$$

where $R_{\varphi,t}$ denotes the return of portfolio φ and λ is the investor's risk aversion. To assess the perceived outperformance of the GARCH-RB portfolio over the standard GARCH-based ERC portfolio, we compute the fee an investor would be willing to pay to capture the utility gain from investing in the range-based portfolio. This amounts to find Δ such that

$$\sum_{t=1}^{T} U(R_{\text{GARCH},t}) = \sum_{t=1}^{T} U(R_{\text{GARCH-RB},t} - \Delta).$$

Figure 2 reports the annualised value of Δ for different levels of risk aversion for the GARCH-RB portfolio built on five-day forecasted covariance matrices on our five assets investment universe. To check robustness, we also compute this metric for less diversified portfolios, composed only of bond futures and equity indices futures from a single geographic zone (US or Europe). For all three portfolios, the implied fee is positive, highlighting the benefits of including range-based volatility measures in the construction of range-based portfolios. Looking at various levels of risk aversion parameters, the magnitude of this fee is around 10 basis points. Although the utility gain may seem low, it comes as an effortless premium as range-based measures are readily available and come at a limited cost.

FIG. 2 ANNUALISED UTILITY GAIN FOR DIFFERENT LEVELS OF RISK AVERSION



Source: LOIM. For illustrative purposes only.

[4] Quadratic utility function focuses on the first two moments of the portfolio returns distribution, a standard assumption in the mean-variance paradigm of portfolio construction.

Conclusion

The first two editions of MARS were devoted to assessing the statistical properties of range-based volatility measures and evaluating their performance in forecasting the path of the unobserved volatility process. Although important from a statistical point of view, these questions are arguably of lesser interest to investors if they are not linked to actual outperformance at the portfolio level.

In this white paper, we highlight how range-based volatility measures can be leveraged to build better performing risk-based portfolios. In the particular context of Equal Risk Contribution, we find that a risk-averse investor would prefer to invest in a portfolio incorporating intraday data, emphasising that the advantages of range-based measures are not purely statistical but meaningful in a portfolio construction exercise. This result concludes our triptych on range-based volatility measures and our series will now turn to the identification of macroeconomic regimes and the efficiency of macro-risk-based portfolios.

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