LOMBARD ODIER
INVESTMENT MANAGERS

Investment viewpoint

Our approach to factor investing

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Nuts and bolts of factor investing

The broad definition of factor investing is the exposure to common return drivers through well-diversified portfolios of stocks. For many years, factor investing has been known mainly in the form of passive replication of market indices. The idea of index replication is rooted in the Modern Portfolio Theory that advocates a single factor view of the stock market. According to this theory, market risk is the only rewarding factor, while the portfolio collectively held by the market is the most efficient replication of this factor.

The unique role of this market factor has been challenged from the very beginning. However, it took a long time before the existence of multiple rewarding factors was widely accepted, both in academia and the industry. Over time, practitioners have ventured even further by questioning the efficiency of cap-weighted indices in the first place, and proposing alternative stock weighting schemes based on fundamental or risk-related measures. At LOIM, we used to share this view, offering a range of smart beta solutions formed around the concept of equal risk contribution.

These days the financial industry displays numerous solutions for factor investing. Those solutions vary noticeably in terms of factor definitions and portfolio construction methodologies. Factor definitions used in real-life portfolios tend to be more sophisticated than the ones found in academic literature. We observe that this can be explained by the difference in objectives. While the academic research is focused primarily on proving the existence of a factor premia, practitioners are looking for a more robust performance. In practice, the choice of factor definitions is influenced by multiple considerations such as academic coverage, evidence of past performance, common sense and even individual convictions and preferences.

Surprisingly, portfolio construction methodologies seem to be as diverse as factor definitions. The debate on the most efficient approach to implementing factors in portfolios has intensified over the last few years. Practitioners' journals are flooded with papers advocating various methodologies that can be broadly classified as top-down or bottom-up. A top-down approach starts from building single factor portfolios, and then uses these as building blocks for a multi-factor portfolio. A bottom-up approach first integrates factors into a single composite score and then builds the final portfolio based on this single metric. There is conflicting evidence of the superiority of one solution over the other, which therefore implies that implementation details would seem to play an important role.

In this paper we present our view on how to best incorporate factors in equity portfolios. After reviewing the variety of existing methodologies, we decided to take a "scientific" path by clearly defining the portfolio objective and detail the model for stock returns. Such a disciplined approach allowed us to come up with a clear-cut methodology that we believe implements factors in the most efficient way. The structure of the paper reflects our "chemin de pensée" or "way of thinking." We tried to keep the discussion less technical or academic as possible and rather focusing on practical applications, putting additional details in footnotes and all the formulas and derivations in the Appendix.

Back to basics

Our starting point is the classical multifactor framework, where stocks' returns are driven by a linear combination of common factors plus an independent stocks' specific component:

$$\text{Stock return} = \text{Market} + x_1\,\text{Factor}_1 + \cdots + x_m\,\text{Factor}_m + \text{Specific}$$

The key requirement for factor investing is the ability to minimise the unrewarded stocks' specific risk through well-diversified portfolios. Of course, this task can be accomplished only if we have enough stocks in our universe. If this is not the case, we are in the domain of high conviction investing where one must pay attention to single names rather than focus on systematic drivers. With the universe of stocks being large enough we can reasonably expect the performance of well-diversified portfolios to be largely determined by their factor exposures and not their composition. We will maintain this assumption throughout the rest of the paper.

With specific risk being taken care of, the next step is to decide which factors are rewarding and which systematic risk needs to be hedged away. To begin with, we do not have a particular view on the future evolution of the market in general,[1] therefore, we require our portfolios to have zero exposure to the market factor. As all the stocks have the same exposure to the market factor, this condition means that our portfolios must be cash neutral.[2] Other unrewarded factors include geographical and sector exposures, therefore, we constrain our portfolios to be sector and regional neutral.

The success of factor investing depends on a careful selection of factors. We have chosen a conservative approach by accepting only five factors with the strongest evidence of positive premia: Value, Quality, Momentum, Low beta and Small size. Following the standard industry practice, we assume that stocks' factor exposures can be measured explicitly. For example, the exposure to Value is derived by aggregating different price ratios like price-to-book and price-to-earnings and ranking the aggregate measure across stocks. Every portfolio manager chooses a particular way of computing factor exposures including the choice of financial ratios used in the construction and types of data transformations. In this paper, we will not discuss methodologies of measuring factor exposures, and focus primarily on how to turn those exposures into portfolio weights.

We are now set up for the discussion of an optimal portfolio construction. We begin with an ideal case of an unconstrained long-short portfolio before highlighting the challenges of real-life solutions.

Bottom-up or top-down?

A rational investor seeks to maximize expected reward per unit of risk. To build portfolios with the best trade-off between risk and return (Sharpe ratio), we will apply the classical machinery of the mean-variance optimization.

But first, we need to agree on our expectations about factor premia. This is not an easy task. Some practitioners advocate timing of factors based on their past performance, current valuation or even economic cycles. We are not convinced that factor timing will bring any value when compared to a straightforward diversification, knowing all the pitfalls of such an approach.[3] Therefore, we choose to be agnostic in our convictions that five factors will deliver the same uncorrelated performance on a risk-adjusted basis.

Given our convictions, the optimal portfolio exhibits a very intuitive characterization. It is a minimum risk portfolio with equal risk exposures to rewarding factors and zero to others (see Appendix for further details). Since, by assumption, all the factors have an equal likelihood to deliver excess performance per unit of risk, the optimal portfolio should naturally allocate an equal risk budget across the systematic drivers. Amongst all the portfolios that share this property and magnitudes of factor exposures, we prefer the most diversified one as the stock specific risk is not rewarded.

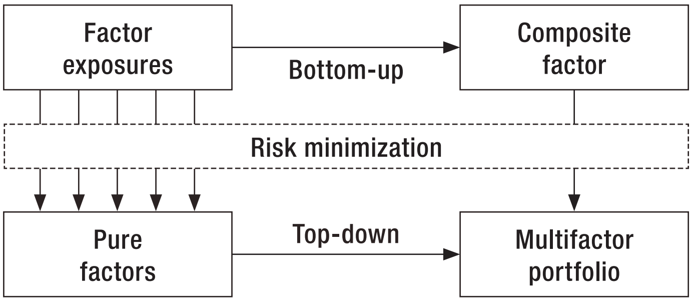
The risk-based approach to factors requires the knowledge of factor risk. Although factor returns are not directly observable, they can be approximated by returns of portfolios that target single factor exposures. The most efficient factor replication is achieved by *pure factor portfolios* (or simply pure factors), which are minimum risk portfolios with a unit exposure to the corresponding factor and zero to others.

A remarkable feature of the optimal multifactor portfolio is that it represents a simple mix of pure factors (see Appendix), meaning that the portfolio construction is effectively reduced to combining pure factor portfolios with weights being equal to the desirable factor exposures. Whatever the choice of factor exposures is, the building blocks are the same. In practice, such a portfolio construction is often called a top-down approach.

An alternative to top-down portfolio construction – the bottom-up methodology – consists of aggregating factor exposures into a single composite factor on the security level, and then building a portfolio from the composite factor only. It appears that the top-down portfolio can be equivalently built using the bottom-up approach via an appropriate method of factor aggregation, which takes into account factor interactions[4] (see Appendix). The equivalence between bottom-up and top-down portfolio constructions is schematically illustrated in Exhibit 1.

[1] Any such conviction should be expressed within an asset allocation framework.
[2] Long and short parts have the same dollar value. We discuss beta-neutral portfolio later in the paper.
[3] See Bender et al. (2018) for the referred discussion.
[4] Cross-sectional correlations between stocks' factor exposures.

EX. 1 TWO APPROACHES TO PORTFOLIO CONSTRUCTION



Source: LOIM. For illustrative purposes only.

This result makes the debate on the pros and cons of the two competing approaches somewhat pointless and this debate has also been recently intensified in practitioner's journals.[5] We have observed that the majority of authors seem to believe that the bottom-up approach is more efficient as it copes better with factor interactions. Our analysis however shows that the two constructions are fully equivalent, provided that they are implemented in an optimal way: top-down portfolio construction must mix pure factor portfolios, while the bottom-up approach must use the composite factor that takes into account factor interactions.

Statistical view of pure factors

By construction, returns of pure factor portfolios most accurately replicate returns of factors that are otherwise unobserved. Alternatively, factor returns can be estimated statistically by running a cross-sectional regression of realized stock returns on their factor exposures, which is a common practice in academic research since the work of Fama and MacBeth (1973). Each coefficient of such multivariate regression will represent an estimate of the excess return generated by a unit exposure to a given factor, other things held constant.

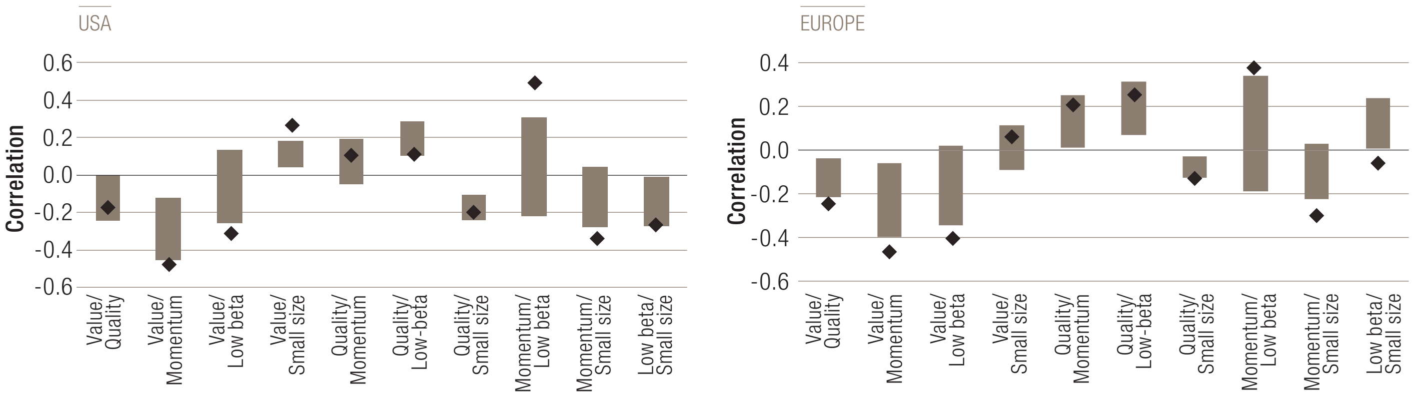
It is probably unsurprising that factor returns estimated statistically are identical to returns of pure factor portfolios[6] (Melas et al 2010). Clarke et al. (2017) follow a slightly different path by incorporating weights of the market index into the regression estimation, thus putting more emphasis on explaining performance of larger cap stocks.[7] Pure factors built in this way will have a higher concentration of weights in larger caps, which has several advantages. First, pure factor portfolios are better suited for building long-only portfolios with the objective to outperform a cap-weighted index. Second, larger cap stocks are more liquid meaning that transaction costs might be lower.

We are not big fans of incorporating market weights into factor construction. In our opinion, the concentration in large caps reduces the breadth of the systematic strategy. Our analysis shows that pure factors built this way have generally inferior performance to the ones built in a "benchmark-agnostic" manner. Furthermore, liquidity is typically not an issue since our stock universe includes primarily large and mid-caps. Regardless, we prefer to use an optimization-based procedure for portfolio construction that incorporates all the relevant constraints as discussed later in the paper.

Importance of factor interactions

Pure factors are natural building blocks for portfolio construction as each portfolio is exposed to only one factor being "cleaned" from the secondary effect of other factors. Exhibit 2 illustrates the importance of taking into account factor interactions. For two major regions USA (S&P 500) and Europe (MSCI Europe), we show standard deviation bounds of cross-sectional correlations between stocks' factor exposures computed at the end of each month from December 1998 to September 2019. The levels corresponding to end of September 2019 are marked by crosses.

EX. 2 HISTORICAL FACTOR INTERACTIONS: DECEMBER 1998 – SEPTEMBER 2019



Source: LOIM. For illustrative purposes only. Past performance is not indicative of future results.

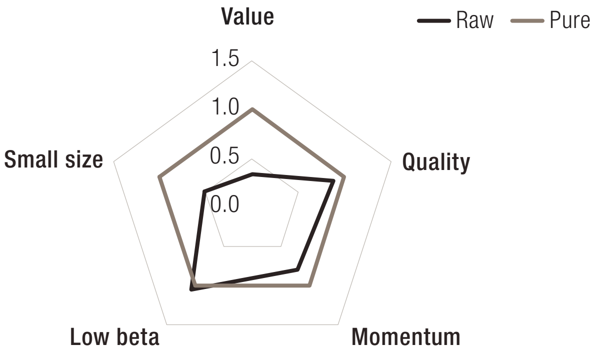
5 See Chow et al. (2018) for the literature review.

6 Strictly speaking, we should estimate a generalized least-squares taking into account the specific risk covariance matrix.

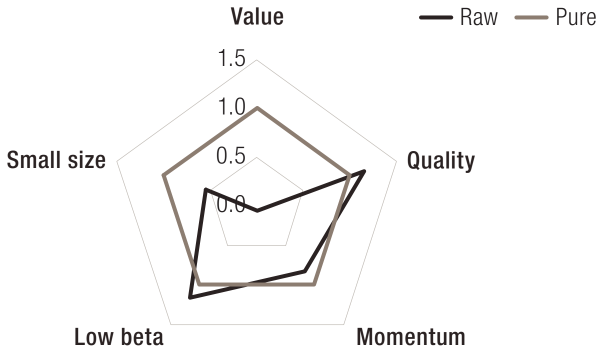
7 This is equivalent to assuming that stocks' specific variances are inversely related to their market capitalizations. Such definition of pure factors is used, for example, by Bloomberg.

EX. 3 FACTOR EXPOSURES OF TOP-DOWN PORTFOLIOS AS A MIX OF PURE AND RAW FACTORS: AS OF SEPTEMBER 2019

USA



EUROPE



Source: LOIM. For illustrative purposes only. Past performance is not indicative of future results.

We observe a strong negative interaction between Value and other factors, which has fundamental reasons. To begin with, Value and Momentum are naturally negatively correlated since stocks with negative price momentum tend to be cheap. In particular, the current correlation between the two factors is very negative when compared to historical levels due to the recent disappointing performance of Value everywhere. Value is also negatively correlated with Quality as good quality stocks tend to be expensive.

Looking at other interactions, we note that low-beta stocks are typically rich in valuations and of good quality. Most recently, they are particularly expensive relative to historical levels. Low beta and Momentum have overall the most unstable correlation, whose sign and magnitude depend on the direction and strength of the past market performance. During periods of the stock market rally, low beta stocks are lagging behind, therefore, they tend to have negative exposure to momentum. The magnitude of correlation between the two factors becomes positive following a period of market sell-off.

It is insightful to compare pure factors with *raw factors* that are built by optimizing on single factor exposures without accounting for factor interactions.[8] Exhibit 3 shows factor exposures of top-down portfolios built by mixing pure factors and raw factors.

By construction, the portfolio built from pure factors has the same unit exposure to each factor. In contrast, with raw factors as building blocks, factor exposures become imbalanced as can be observed in Exhibit 3.

Firstly, the average factor exposure is lower as secondary exposures tend to partially offset the primary ones. Secondly, raw factor portfolios are strongly underexposed to Value, while being overexposed to Low Beta. Underexposure to Value is not unexpected as this factor is mostly negatively correlated with the others (as shown in Exhibit 2). The danger of ignoring interactions between Value and other factors is that multifactor portfolios may end up overexposed to crowded factors, which is the case of the Low Beta factor presently.

Given the scale of factor interactions, it is worth noting that single factor indices offered by MSCI and Russell index are all built “raw” i.e. without accounting for interactions.[9] For multiple factor exposures, both MSCI and Russell Index advocate a bottom-up approach. The composite factor used in the construction of MSCI indexes is a simple average of single factor scores without accounting for factor interactions. Russel advocates a so-called “multiple-tilt” methodology[10] that allows reducing the effect of negative interactions between factors, however, it does not fully address the problem of balancing factor exposures.

Anatomy of factor premia

Our analysis above relied on a linear multifactor model, which postulates that portfolio expected returns are proportionate to their factor exposures. However, knowing that factor exposures are computed in a largely adhoc fashion, this seems to be a rather strong belief.

For example, let’s say that the excess performance of a factor is generated mostly from shorting 20% stocks with the lowest factor exposure while the rest 80% do not perform differently. If this is the case, our pure factor portfolios built on the linear model will not harvest the factors’ premia in an efficient way. The potential non-linear relationship between exposures and returns has also an important implication for real-life portfolios that tend to be constrained on the short side. For example, if factor performances come mostly from shorting “worst” stocks than it may not be easily replicable in long-only portfolios.

To test the validity of the linear model specification, we compare performances of three multifactor portfolios.

All three portfolios have exactly the same exposures to factors, however, these portfolios are built on different sets of stocks. The “*Original*” portfolio represents a mix of pure factors built on the full universe of stocks. The “Long Skew” portfolio achieves exactly the same exposures to the five factors as the original portfolio, however, on a reduced universe that excludes one-third of stocks with lowest composite factor score.[11]

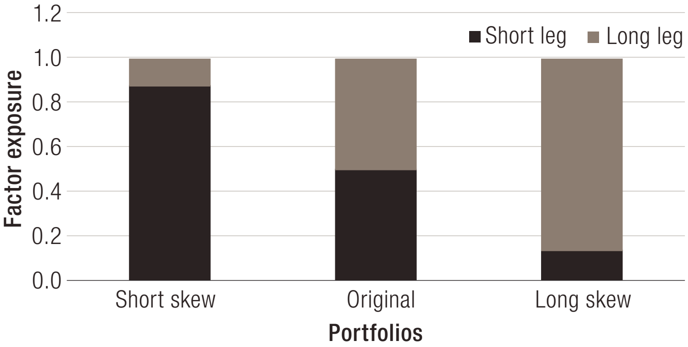
8 A raw factor portfolio is a minimum risk portfolio with unit exposure to the corresponding factor without controlling for exposures to other factors. The return of such portfolio will be identical to a coefficient of a single-variable regression of stocks’ returns on their exposures to the factor under consideration.

9 The methodology of MSCI Enhanced Value Indexes available can be accessed at https://www.msci.com/index-methodology) and FTSE Global factor index methodology at https://www.ftserussell.com/products/indices/factor.

10 Russell (2017).

11 As we established earlier, the top-down portfolio construction is equivalent to the bottom-up one. This means that there exists a composite score that leads to the optimal multifactor portfolio through a bottom-up construction. We use this implied score for the construction of “skewed” portfolios.

EX. 4 SOURCES OF FACTOR EXPOSURE: SHORT VERSUS LONG LEG



Source: LOIM. For illustrative purposes only. Past performance is not indicative of future results.

To maintain the same factor exposures, this portfolio will have to be "skewed" on the long side towards stocks with the highest exposures to factors. And lastly, in a similar fashion, the "*Short Skew*" portfolio is built after excluding one-third of stocks with the highest composite factor score.

Exhibit 4 illustrates different ways the three portfolios achieve the same factor exposure. For the illustration, we use a simple example with a single factor with uniformly distributed exposure across stocks. Total factor exposure of each portfolio is a sum of its short and long leg exposures. In the *Original* portfolio, both legs equally contribute to the total result. The Long Skew portfolio has the same factor exposure as the Original portfolio, however, it achieves it in a different way. Due to a reduced efficiency of shorting stocks, the long leg becomes the dominant source of the portfolio exposure to the factor. This picture is fully reversed for the Short Skew portfolio.

Exhibit 5 compares the historical performance of three multifactor portfolios for the two major regions. Note that volatilities of "skewed" portfolios are expectedly higher as the risk diversification is weaker on reduced universes of stocks. The realized returns of the three portfolios are different as well, meaning that the linear multifactor model may not be the most accurate description of the stock market. We notice however that "long-skew" portfolios seem to outperform. This is good news for long-only portfolios where the short side is heavily constrained.

Redefining the market factor

Thus far we have been dealing with cash-neutral portfolios, which are characterized by the same dollar value of the short and the long side. The reason why we decided to focus on a cash-neutral solution from the beginning is straightforward: as we do not have any view on the market factor, our portfolio should have no exposure to it.

In practice, market exposures are traditionally measured by stock betas (which are statistical sensitivities of stock returns to that of a certain index). As long as betas are not the same across stocks, beta neutrality is not the same as cash neutrality. Stock betas can be directly incorporated into our multifactor model by replacing the unit exposures to the market factor with betas. Repeating the same analysis as before, we can demonstrate that the optimal portfolio represents again a mix of beta-neutral pure factors.

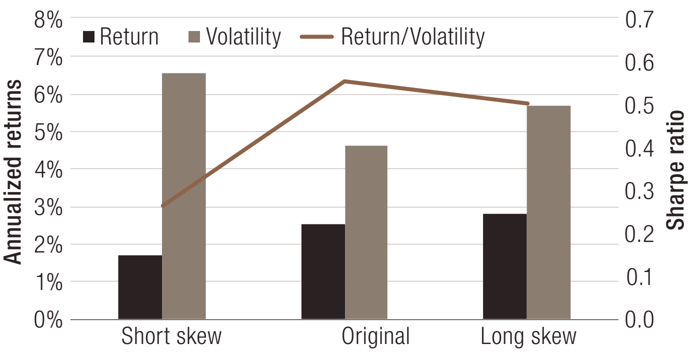
It is important to emphasize that risk characteristics of beta-neutral factors will be different from their cash-neutral counterparts. One can speculate whether factor allocations should be based on their cash-neutral or beta-neutral volatilities. In our view, this consideration will mostly affect the low beta factor whose beta-neutral volatility is much lower than its cash-neutral one. The academic way of replicating low beta factors involves leverage since the existence of this premia is originally explained by the inability of most investors to do so.[12] Keeping this in mind, we are in favor of determining factor allocations on the basis of their beta-neutral risk characteristics.

Factor investing with long-only portfolios

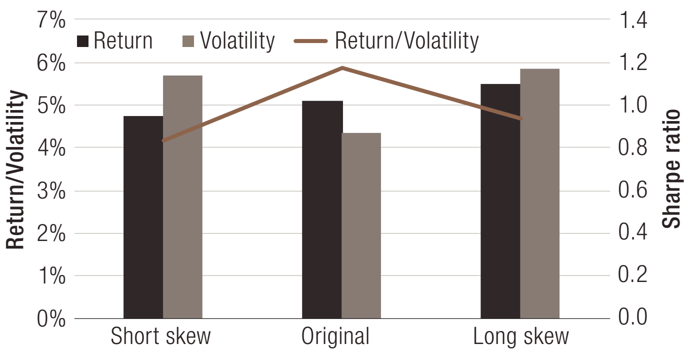
In previous sections we described an ideal world where we can short any stocks in arbitrary quantities. In practice, shorting can be difficult due to cost and access considerations or simply portfolio restrictions. Below we will discuss the most restricted case of long-only portfolios that are constrained to have only non-negative stock weights.

EX. 5 TEST OF THE LINEAR MODEL SPECIFICATION

USA



EUROPE



Source: LOIM. Based on monthly rebalancing from December 1998 to September 2019.
The 'Original' Portfolio represents a mix of pure factors with a 1% risk budget. For illustrative purposes only. Past performance is not indicative of future results.

[12] Black (1972) first described the version of the Capital Asset Pricing Model (CAPM) with a limited access to borrowing, which gives rise to zero-beta factors. Frazzini and Pedersen (2014) extend his work and find a positive premia of the Bet-Against-Beta factor built in a beta neutral way.

A long-only portfolio is built with the purpose of outperforming a certain benchmark, which is typically given by a market index. Therefore, the excess return of this portfolio over its benchmark is the most relevant performance metric. Such excess performance is identical to the return of a long-short strategy that goes long this portfolio and short the benchmark. Therefore, long-only portfolios can be viewed as special cases of long-short portfolios where the stocks' weights on the short side are constrained to not exceed their weights in the benchmark. In our discussion of long-only portfolios we will follow this interpretation.

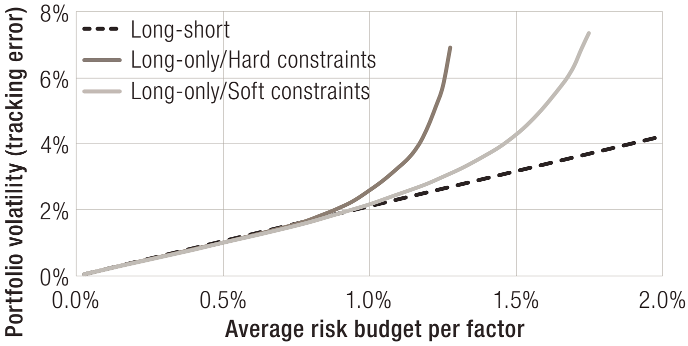
An efficient long-only multifactor portfolio solves the risk-minimization program with an addition of long-only constraints. The optimal solution of such an optimization will be different from the unconstrained one discussed before in two important ways. First, it is no longer available in an analytical form thus requiring a proper numerical optimization. Second, the multifactor portfolio is not a mix of long-only pure factor portfolios[13] anymore, meaning that it cannot be built using a top-down approach. The reason is very simple: imposing long-only constraints on the global portfolio level is less restrictive than at the single factor level.

Being less efficient, a top-down approach will result in a portfolio that is riskier than the optimal one for the same factor exposures. In fact, moderate exposures to factors may no longer be feasible through the top-down construction, while still achievable through an optimization process. Some researchers argue that the top-down approach leads to an inferior result due to factor interactions. In our view, this is not the main reason, as those interactions are appropriately managed with pure factor portfolios.[14] Despite its apparent weaknesses, the top-down portfolio construction has one important advantage: it provides a full transparency of the contribution of each factor to the final portfolio. The benefit of this transparency is appreciated by some practitioners that prefer mixing factor portfolios.[15]

If we are to choose between top-down and bottom-up approaches, in our view the bottom-up one seems to be a better alternative. However, it has its own drawbacks. A generic bottom-up approach forms the portfolio based on a composite factor. As we emphasized in previous sections, factor weights in the composite factor should reflect factor interactions to make sure that the final portfolio has a balanced exposure to factors. In the long-only case, the factor weighting scheme must furthermore account for factor biases introduced by portfolio constraints,[16] which are difficult to assess before the actual portfolio is built. Knowing the limitations of top-down and bottom approaches, we opt for a proper optimization solution that allows achieving a balanced factor exposure in the most efficient way.

Whatever the choice of portfolio construction methodology, long-only portfolios have a limited capacity to generate factor exposures. Indeed, in the long-short world, higher factor exposures are achieved simply by leveraging the optimal portfolio, whereas in the long-only case we have to concentrate the portfolio in stocks with higher factor exposures, which might eventually impact the portfolio diversification. Pushing factor exposures beyond certain limits could result in a rise of portfolio risk due to the stock's specific component, which will no longer justify the gain in expected performance.

EX. 6 IMPACT OF FACTOR EXPOSURES ON THE RISK OF PORTFOLIO



Source: LOIM. Illustration based on actual data.
For illustrative purposes only. Past performance is not indicative of future results.

This point is illustrated in Exhibit 6, which shows a typical relationship between the average risk budget allocated to factors and the portfolio tracking error. Being a simple mix of pure factors, the total risk of a long-short portfolio is comprised only of systematic risk.[17] Therefore, a higher risk budget per factor translates into a proportionate increase in the portfolio risk as shown by the straight line. Since, by assumption, the expected return of each factor is proportionate to its risk budget, the slope between this line and the vertical axis can be interpreted as the portfolio's Sharpe Ratio.

The risk of the long-only portfolio tracks the risk of the long-short portfolio up to roughly 2% tracking error. Pushing the exposure to factors beyond this point results in a degradation of the Sharpe Ratio as the stocks' specific risk starts to be important. In Exhibit 6, this is evident by an increasing curvature of the curve. For example, the portfolio with 3% tracking error has just a 20% higher exposure to factors than the portfolio with a 2% tracking error. In our example, a risk budget in excess of 1.5% per factor is not compatible with long-only constraints at all.

Some factors are relatively less impacted by long-only constraints than the others. For example, long-only portfolios tend to have large capacity to generate Small Size factor exposures since their benchmarks are typically cap-weighted indexes. Furthermore, since large-cap companies tend to be of good quality (refer to Exhibit 2), the tilt to the Quality factor is typically the most difficult to generate. Value is another "difficult" factor due to its tendency to be negatively correlated with other factors.

Taking these considerations into account, the requirement of equal risk budget across factors may not be an optimal choice for higher tracking error portfolios. Imposing a soft risk budgeting seems to be a better solution. As an example, Exhibit 6 shows the risk/return curve of a long-only portfolio where factors' risk budgets are allowed to deviate 30% from the average. At low levels of risk, the slope of the curve does not change meaning that the factor risk is still well diversified. With tracking error getting higher, the curvature does not increase as quickly as before due to a lesser impact of the stocks' specific risk. Clearly, this solution is more suitable, for example, for a 3% tracking error target.[18]

13 A long-only pure factor portfolio is a minimum risk portfolio with a target exposure to the corresponding factor and zero to others. It is important to note that the composition of a long-only pure factor portfolio will depend on the choice of the target exposure.

14 Blitz and Vidojevic (2019) show that the difference in performances between top-down and bottom-up portfolios disappears after matching factor exposures of the two portfolios. Leippold and Rueegg (2018) argue that the two approaches do not perform in a statistically significant way when proper tests are applied.

15 Li and Steidl (2018) discuss the practical merits of the top-down construction. Amenc et al. (2017) describes a top-down approach with a bottom-up enhancement.

16 Exposure to Small Size tends to be exaggerated as large cap stocks are less constrained on the short side.

17 We identify the risk of the pure factor portfolio with the risk of the factor.

18 Target performance/risk represents a portfolio construction goal. It does not represent past performance/risk and may not be representative of actual future performance/risk.

Let us briefly recap the two main challenges of factor investing in long-only portfolios. First, the top-down and bottom-up portfolio constructions are no longer equivalent. Both approaches being sub-optimal, we opt for a portfolio construction based on optimization. Second, high tracking error objectives may result in poor factor replication due to insufficient diversification of stocks' specific risk. This effect can be partially mitigated by adopting a soft factor risk budgeting.

Inside the black box of optimization

Real-life portfolio construction involves multiple constraints that control different characteristics of portfolio other than factor exposures. The optimization approach is best suited for this task as it allows us to achieve desirable factor exposures in the most efficient manner, while satisfying all those constraints. Although the optimization itself is technically straightforward, it is eventually difficult to understand how various constraints impact the resulting portfolio and how they contribute to the performance. Even without constraints, separating the contribution of factors is not obvious. This is why optimization-based solutions are often considered as a black-box, and avoided in favor of less efficient but more transparent solutions.

Fortunately, we have at our disposal a straightforward methodology that allows us to decompose any optimized portfolio into a sum of single factor portfolios as well as portfolios accounting for the pure impact of each constraint.[19] In general, there are many ways to do it. For example, we could use pure factor portfolios as a basis for decomposition and then split the residual difference equally between factors assuming no impact of constraints. In our view, this is not a good solution as it masks the contribution of each portfolio constraint on the overall portfolio and on each factor in particular.

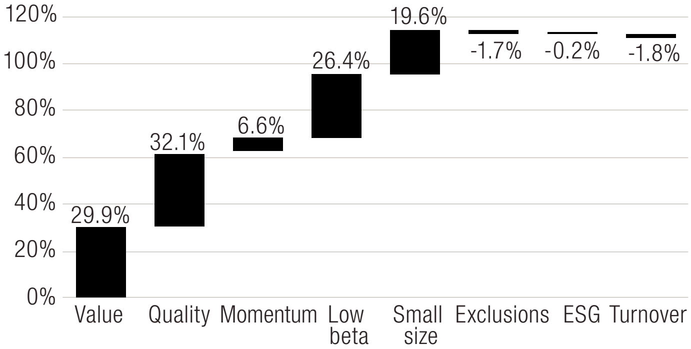
The idea behind our approach is to perform the portfolio construction in small increments. At each step, we run an optimization procedure that targets a certain fraction of the target factor exposures under milder constraints. With a sufficiently small increase in factor exposures and the strength of constraints, the effect of factors and constraints on portfolio weights becomes additive: the incremental portfolio will be a simple sum of single factor portfolios and portfolios that account for the effect of each constraint. By construction, the final portfolio is the sum of incremental portfolios, therefore, its decomposition is obtained by aggregating decompositions of the incremental portfolios (see Appendix for more details).

As an illustration, Exhibit 7 shows the performance attribution of our ARP Equity Market Neutral Strategy. Apart from factor exposures, the portfolio respects exclusion lists (as determined as LOIM policy) by not taking exposures in certain stocks. Furthermore, the long side of the portfolio is required to have a better ESG score than the short side,[20] and its turnover is controlled by adding a penalty to the risk objective. The performance of the optimized portfolio can be fully attributed to all the elements of portfolio construction including the impact of turnover control.

Key takeaways

In this paper we described our view on the efficient multifactor portfolio construction. We first stressed the importance of diversification of factor premia. In our view, this can be achieved through an equal-risk allocation between rewarding factors while ensuring zero exposure to non-rewarding ones. In the absence of portfolio constraints, this can be accomplished through a top-down construction with pure factor portfolios as building blocks.

EX. 7 PERFORMANCE ATTRIBUTION OF ARP EQUITY MARKET NEUTRAL STRATEGY



Source: LOIM. From December 1998 to September 2019. Based on monthly rebalancing gross of transaction costs without compounding. For illustrative purposes only. This material contains hypothetical (simulated) backtested performance results and other related information ("Hypothetical Results"). The period shown for the Hypothetical Results is based on available information and LOIM believes the period to be representative and statistically valid. Changes in the assumptions would have a material impact on the Hypothetical Results and other statistical information based on the Hypothetical Results. Past performance is not indicative of future results. These performance results are backtested based on an analysis of past market data with the benefit of hindsight, do not reflect the performance of any LOIM product and are being shown for informational purposes only. While the results presented are based on certain assumptions that are believed to reflect actual trading conditions, these assumptions may not include all variables that can affect, or have affected in the past, the execution of trades. The hypothetical portfolio results are based on the following assumptions: (1) The hypothetical portfolio record does not include deductions for brokerage commissions, exchange fees, or slippage; (2) It assumes purchase and sale prices believed to be attainable. In actual trading, the prices attained may or may not be the same as the assumed order prices; (3) The portfolio results do not take into account any tax implications arising from the sale or purchase of securities, which in actual trading do have an impact on gains and losses.

In real life, portfolio restrictions such as, for example, long-only constraints, make the straightforward top-down approach inefficient as optimal building blocks are not unique anymore. We insisted on the need to use a numerical optimization procedure that achieves an equal-risk exposure to factors with minimal idiosyncratic risk. A straightforward process allows to characterize the optimal portfolio as a mix of pure factor portfolios thus facilitating the analysis of its performance.

In the paper we further touched upon several practical issues related to long-short and long-only portfolio construction. In the long-short case, we argued that shorting market indexes does not reduce factor premia, and only affects the portfolio risk due to less diversification. This means that shorting stocks, which tends to be costly, can be generally avoided without a sacrifice in performance.

In the long-only case, we emphasized a limited capacity to replicate factor exposure due to the no-leverage constraint. Introducing strong factor tilts in long-only portfolios may result in an adverse impact on the diversification, which means an increase in the tracking error not compensated by a higher expected return. We concluded that a careful selection of the magnitude of factor exposures is an important part of long-only portfolio construction.

[19] Strictly speaking, we decompose the long-short portfolio where the long leg is the portfolio while the short leg is the benchmark.

[20] Accounting for the difference in dollar value of the long and the short leg.

What next…

The success of factor investing will eventually depend on the persistence of factor premia. Recent disappointing performance of factors, especially Value, intensified the debate on the future of factor premia and merits of factor timing. Advocates of factor timing insist that it is possible to add value by dynamically adjusting allocations to factors based on momentum signals, factor valuations and even the state of economy. Their opponents, however, claim that factor timing reduces the benefits of diversification while incurring additional transaction costs.[21]

Factor investing relies on only certain factors that have been discovered in the past, whose future as a source of excess performance is a matter of hot debate. At the same time, new factor premia are not easily accepted by the community on reasonable concerns around the risks of data mining.

We believe that this impasse can be overcome by making the process of factor discovery truly systematic, and as much as possible free from human biases. Methods of machine learning are slowly but firmly penetrating the world of systematic equity, which may eventually add the missing momentum to factor investing.

In our view, applying machine learning techniques to a wide set of stocks' attributes including alternative data such as ESG will serve two main purposes. First, it will allow us to discover new sources of alpha either through exploring non-linear combinations between stocks' attributes not captured by traditional factors or leveraging on new data sources. Second, the dynamic nature of machine learning will make the systematic process adaptive to changing market environment. This application of machine learning to systematic equity is the focus of our current research efforts.

[21] For example, Gupta and Kelly (2019) find that the factor momentum generates economically significant returns. While Dichtl at al. (2019) claim that such strategies do not yield significant performance after transaction costs.

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Appendix

Characterization of the maximum Sharpe ratio portfolio

Let X be a $n \times m + 1$ matrix of n stocks' exposures to $m + 1$ factors. The first column of X contains units everywhere, which are stocks' exposures to the market factor. Next we denote by Σ_f a diagonal factor covariance matrix, and by Σ_ϵ stocks' specific covariances. The full covariance matrix of stocks is a sum of systematic and specific risks:

$$\Sigma = X\Sigma_f X' + \Sigma_\epsilon$$

Using this specification, one might be tempted to estimate factor covariances from historical covariances of stocks. However, this is not methodologically possible since stocks exposures to factors are not constant over time. Therefore, we should interpret the risk model above as forward looking reflecting our views on future factor performances.

Consider an optimization problem that seeks a minimum risk portfolio with given exposures to factors:

$$\min \mathrm{w}'\Sigma\mathrm{w}, \ \ \mathrm{s.t.}\, X'\mathrm{w} = e$$

where e is the vector of target risk exposures. Since factor risk is fixed by constraints on factor exposures, we can replace the total risk by the specific risk in the objective function. Using the standard technique of solving optimization problems, it can be verified that the optimal solution is given by:

$$\mathrm{w} = \ \Pi e$$

where Π is a $n \times m$ matrix with i-th column being a pure factor having a unit exposure to i-th factor and zero to others:

$$\Pi = \Sigma^{-1}X(X'\Sigma^{-1}X)^{-1} = \Sigma_\epsilon{}^{-1}X(X'\Sigma_\epsilon^{-1}X)^{-1}$$

As pure factors are well-diversified portfolios by construction, their covariances should be close to the covariances of actual factors.[22] Using formula for pure factors, we derive the following approximation:

$$\Sigma_f \approx \Pi'\Sigma^{-1}\Pi = (X'\Sigma^{-1}X)^{-1}$$

According to our conviction, five rewarding factors have the same risk-adjusted expected returns, which, without loss of generality, can be set to unity. Furthermore, since factor returns are independent, we can assume that other non-rewarding factors have zero expected returns. As we will see it below, this assumption guarantees that the portfolio will have no exposure to unrewarded factors.

Our views on risk-adjusted returns of factors can be summarized by a vector v, whose elements are either unity (rewarding factors) or zero (unrewarding factors). Using this notation, we can write expected returns of factors as $\Sigma_f{}^{\frac{1}{2}}\mathrm{v}$, and expected return of stocks as $X\Sigma_f{}^{\frac{1}{2}}\mathrm{v}$. The optimal mean-variance portfolio is equal to the product of the inverse covariance matrix and the vector of stocks' expected returns:

$$\mathrm{w} = \Sigma^{-1}\left[X\Sigma_f{}^{\frac{1}{2}}\mathrm{v}\right]$$
$$= [\Sigma^{-1}X(X'\Sigma^{-1}X)^{-1}](X'\Sigma^{-1}X)\Sigma_f{}^{\frac{1}{2}}\mathrm{v} = \Pi(X'\Sigma^{-1}X)\Sigma_f{}^{\frac{1}{2}}\mathrm{v} \approx \Pi\left[\Sigma_f{}^{-\frac{1}{2}}\mathrm{v}\right]$$

The last inequality is established by using the approximation of factor covariances derived above. This results shows that the optimal portfolio is a mix of pure factors with zero weights for non-rewarding factors and equal risk allocation between rewarding ones.

The formula for pure factors involves stocks' specific covariances Σ_ϵ. In practice, there are several ways to specify it. A simple and robust solution is to assume an identity matrix, which automatically results in a cash-neutral portfolio. A slightly more sophisticated approach is to use a diagonal matrix of stocks variances. Although those variances represent a combination of systematic and specific risk, one can argue that on the stock level the specific risk is dominating.

Equivalence of top-down and bottom up portfolio construction

We have demonstrated that optimal portfolio is a top-down combination of pure factor portfolios. Let us now show that the optimal portfolio can be also built bottom-up.

Let e be some generic factor weights used to build a top-down multifactor portfolio:

$$\mathrm{w}^{\mathrm{t/d}} = \Pi e = \Sigma_\epsilon^{-1}X(X'\Sigma_\epsilon^{-1}X)^{-1}e$$

Suppose now that we want to represent the systematic risk with a single composite factor keeping the specific risk unchanged. Let us define a vector of composite factor exposures as a weighted sum of individual factor exposures $X\theta$ with weights given by $\theta \equiv (X'\Sigma_\epsilon^{-1}X)^{-1}e$. This weighting scheme takes into account factor interactions as the matrix in parenthesis can be interpreted as a weighted cross-sectional correlation between factor exposures.[23]

The bottom-up portfolio is a minimum risk portfolio with a unit exposure to the composite factor. Replacing X with $X\theta$ in the formula for the top-down portfolio and setting the target exposure to unity, we have the following bottom-up solution:

$$\mathrm{w}^{\mathrm{b/u}} = \Sigma_\epsilon^{-1}X\theta(\theta'X'\Sigma_\epsilon^{-1}X\theta)^{-1} = \mathrm{w}^{\mathrm{t/d}} \times (\theta'X'\Sigma_\epsilon^{-1}X\theta)^{-1}$$

Note that the expression in brackets is a scalar, which does not affect relative weights.

Performance attribution

To simplify the exposition, we will ignore portfolio constraints apart from factor exposures and the long-only constraint. The analysis can be generalized to incorporate any types of additional constraints.

Active weights of an optimal long-only portfolio can be written as a function of m target factor exposures:

$$\mathrm{w} = W(x_1, x_2, \dots, x_m)$$

Consider a grid of values $\{t_i\}_{i=1}^N$ such that $\{t_i\}_{i=1}^N$, $t_N = 1$, and $t_i < t_j$ for $i < j$, and a sequence of optimal long-only portfolios:

$$\mathrm{w}_i = W(t_i x_1, t_i x_2, \dots, t_i x_m)$$

By construction, w_0 is a portfolio with zero active weights, and $\mathrm{w}_N = w$.

22 As we mentioned in the text, the fundamental assumption behind factor investing is that performance of well-diversified portfolios is driven largely by the systematic risk.
23 This weighting scheme is also discussed in Vaucher and Medvedev (2017).

For sufficiently small $t_{i+1} - t_i$ the difference between two consecutive portfolios can be approximately decomposed into a sum of single factor tilts:

$$\begin{aligned} \mathrm{w}_{i+1} - \mathrm{w}_i = \; & W(t_{i+1}x_1, t_{i+1}x_2, \dots, t_{i+1}x_m) - W(t_i x_1, t_i x_2, \dots, t_i x_m) \\ & \approx W(t_{i+1}x_1, t_i x_2, \dots, t_i x_m) - W(t_i x_1, t_i x_2, \dots, t_i x_m) \\ & + W(t_i x_1, t_{i+1}x_2, \dots, t_i x_m) - W(t_i x_1, t_i x_2, \dots, t_i x_m) \\ & \dots \\ & + W(t_i x_1, t_i x_2, \dots, t_{i+1}x_m) - W(t_i x_1, t_i x_2, \dots, t_i x_m) \end{aligned}$$

Each element of this decomposition is a portfolio with non-zero exposure to one factor and zero to others. By definition we have:

$$\mathrm{w} = (\mathrm{w}_1 - \mathrm{w}_0) + (\mathrm{w}_2 - \mathrm{w}_1) + \dots + (\mathrm{w}_N - \mathrm{w}_{N-1})$$

Therefore, the decomposition of the final portfolio can be obtained by aggregating *N* decompositions of incremental portfolios.

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