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Investment viewpoint

Efficient protection of multi-client portfolios: Ingredients are more important than recipe

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Need to know

- The main challenge for a systematic portfolio protection is the design of the mechanism of re-engaging investment after it has been downsized
- Protection of a multi-client portfolio is challenging since such mechanisms should insure the portfolio performance simultaneously over multiple intervals
- An efficient portfolio protection strategy must strike a balance between the downside protection with upside participation[1]

Portfolio protection over a rolling window

Portfolio protection is widely adopted by investment managers, even if they do not explicitly admit it. It may take the form of a systematic strategy, discretionary process, or an involuntary decision to cut risk when the loss becomes intolerable. The main challenge for a systematic portfolio protection is the design of the mechanism of re-engaging investment after it has been downsized, since missing out on the market upside can be as painful as the portfolio loss.

By their design, derivative instruments seem to offer an ideal solution for portfolio protection. For example, a call option on a stock index allows capturing an unlimited index upside while incurring limited losses in case the index underperforms. Although options are readily available only on standard indexes, they can be replicated, at least in theory, on any portfolio of liquid assets through a dynamic index exposure. Such synthetic solutions are known as Option Based Portfolio Insurance (OBPI).

The major limitation of OBPI is the need to explicitly define the investment period, i.e. the initial and the terminal date of the investment. This is not an issue for a single investor but may be a deal breaker for public funds whose investors have multiple entry points and unclear holding periods. The common practice is to evaluate performance of public funds over rolling time intervals of few years broadly in line with the minimum holding period of investors. Such practice means that the portfolio protection strategy should act on multiple investment periods (i.e. over a rolling window) rather than a fixed one.

In our recent academic paper[2] we described a theoretical basis for optimal portfolio protection over a rolling window, and proposed a numerical solution using methods of reinforcement learning.

1 Capital protection represents a portfolio construction goal and cannot be guaranteed.
2 A. Medvedev "Option-Like Portfolio Insurance Over A Rolling Window: Introduction and Derivation by Reinforcement Learning."

While the theory is relatively straightforward, the main difficulty is in the complexity of the optimal strategy. With the holding period being fixed, OBPI requires only two quantitative inputs: the performance of the underlying index since the initial date, and the time left to the terminal date. Its mechanism can be described by a relatively simple analytical formula. In contrast, with the holding period being flexible, the optimal decision requires the knowledge of all the past asset returns within the length of the holding period. As a consequence, the mechanism is described by a high dimensional function, which is impossible to write down on the paper, and it can only be approximated numerically.

The complexity of the optimal portfolio protection means that it is not as interpretable as OBPI, which can be challenging for applications. In this note, we investigate whether we can adapt existing portfolio protection methodologies to a rolling window with the objective of preserving interpretability while achieving reasonable efficiency.

Short list of candidates

Option-based strategy can be implemented to the portfolio protection over a rolling window in two ways. The first is to implement OBPI sequentially. As soon as the strategy comes to the terminal point it is rolled over to the next period. Such a solution clearly offers a perfect protection of the portfolio over intervals that are matching the option horizon, however it may fail to do so over overlapping intervals. More importantly, investors that enter the fund in the middle of the option lifetime will not be protected as well as the investors that entered close to the roll-over dates.

In our view, a better alternative is to run a number of option-based strategies simultaneously on overlapping intervals. Such design can ensure an equal treatment of investors since for every time interval there is a dedicated live OBPI aligned with it. The downside is that OBPI strategies are not 'synchronised'. Indeed, each one protects only a part of investment over a particular time horizon, therefore, there is no guarantee that altogether they will deliver an effective protection of the overall portfolio.

When discussing portfolio protection methodologies, we cannot avoid mentioning Constant Proportion Portfolio Insurance (CPPI), which is often considered as an alternative to OBPI. CPPI represents a more sophisticated version of a stop-loss mechanism where the investment is gradually scaled down as the portfolio loss approaches a certain floor. This mechanism can be easily adapted to a rolling window.

Consider the following rule that determines the index weight as a function of the portfolio loss:

$$w_t = \text{multiplier} \times (L^{max} - L_t),$$

w_t is the index weight, L_t is the peak-to-date portfolio loss over a window of fixed length,[3] and L^{max} is the maximum admitted loss.

CPPI reacts explicitly to the overall portfolio loss, therefore, it avoids the 'synchronisation' issue of the option-based protection. Importantly, the portfolio loss is computed from the portfolio peak, which makes the strategy focus on the most 'troubling' time interval that starts from the date of this peak. On the downside, CPPI does not incorporate any explicit re-engaging policy after the index weight is substantially reduced effectively leaving the portfolio dwelling in a low risk state until its loss naturally "cools down" over time.

EXHIBIT 1. OVERVIEW OF THREE CANDIDATE SOLUTIONS

STRATEGY	PROS	CONS
OBPI	Derived from the optimal portfolio protection over a fixed investment horizon.	Lack of 'synchronisation' between individual strategies.
CPPI	Explicitly limits the portfolio downside	Does not react to the portfolio performance drag, therefore, may miss on upside opportunities.
Balanced	Explicitly balances downside protection with upside participation	Simplistic formula with no clear indication on how well it protects against the portfolio downside

To address these concerns related to the OBPI and CPPI, we can introduce a third candidate defined by the following simple formula:

$$w_t = \frac{U_t}{U_t + L_t},$$

U_t is the peak-to-date portfolio underperformance relative to the index computed over a rolling window, which characterises the missed upside in the index. According to the formula, the index weight in the portfolio depends on where the largest 'regret' is coming from. If, for example, the current underperformance exceeds the loss ($U_t > L_t$), the index gets over-weighted to reduce the risk of further accumulation of the performance drag.

Similar to CPPI, this mechanism does not have the 'synchronisation' issue of the option-based approach. However, unlike CPPI, it incorporates both absolute and relative performance achieving a balance between upside participation and downside protection. In the rest of this note we will call it a balanced solution to emphasize its key advantage. Exhibit 1 summarises strong and weak points of the three portfolio protection mechanisms.

Picking the winner

To evaluate the three portfolio protection strategies described above, we will consider a simulation based environment rather than actual data. Such an approach will allows us to generate a large number of possible market histories instead of focusing on a single one, and observe possible outcomes of each solution.

[3] Peak-to-date loss is similar to maximum drawdown over the interval except that the end point of drawdown is the current date.

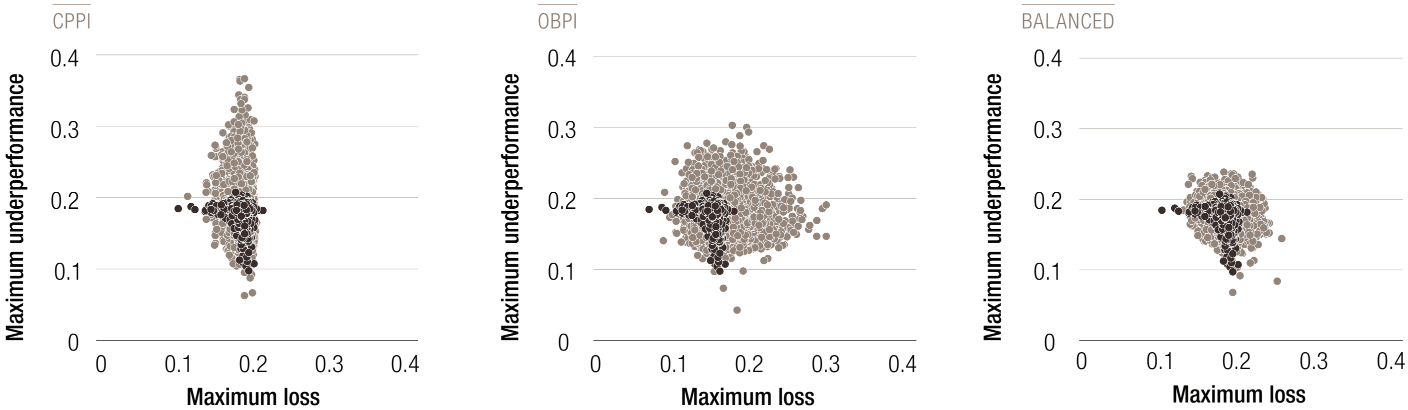
We adopted the numerical example from our academic paper, which was inspired by the characteristics of the S&P 500 index. Specifically, we assume a weekly rebalancing frequency, and postulate that the index over one week can move up or down by the same magnitude of 2%. In the academic literature this type of model is known as the binomial tree model – the workhorse of the option pricing. Of course, the real-life asset returns have more complex properties including an infinite set of possible outcomes, 'fat-tail' distributions, and non-constant volatility. Since no parametric model can be credibly considered as a true descriptor of the real market, we chose the simplest model.

We simulated several thousands market episodes of 20 years each.[4] For each episode, we implemented three strategies[5] introduced in the previous section, and recorded their outcomes. First outcome is the maximum loss of the portfolio over all the 2-year intervals within the episode. Effectively, we run the portfolio over 20 years while measuring its performance over a rolling window of 2 years. The maximum loss characterises the effectiveness of the downside protection of the strategy. The second outcome is the maximum 2-year underperformance relative to the index, which quantifies the missing opportunities – the flipside of downside protection.

Exhibit 2 shows the scatter plot of several thousand outcomes for each of the three portfolio protection strategies (in grey). The spread of the cloud of points gives an idea of how uncertain the outcomes of each strategy are. As a benchmark, we added a scatter plot corresponding to the numerical approximation of the optimal protection over a rolling window described in our academic paper (in black). The distinct angle shape of the outcomes of the optimal strategy obviously catches our attention. It implies that whatever the path of the index is, the portfolio ends up either with a certain level of maximum loss (a flat vertical profile of the shape) or a certain level maximum underperformance (a flat horizontal profile of the shape). The fact is, such behaviour is a reflection of the optimality of the strategy.[6]

Let us comment first on the performance of CPPI.[7] This strategy seems to provide a downside protection on par with the benchmark effectively capping the portfolio loss at the level of 0.2. This does not surprise us given that CPPI is specifically designed with an emphasis on downside protection. However, as evidenced by the vertical spread of observations, CPPI may disappoint enormously missing on the index upside. This is a direct consequence of the lack of an explicit re-engaging mechanism as discussed earlier. It is worth noting that there are still plenty of episodes where CPPI ends with a smaller maximum underperformance than the benchmark. This observation shows the danger of drawing conclusion from historical data, which is only a single point in the cloud. It is a matter of pure luck that CPPI yields a decent result, which can be mistakenly taken as a proof of its efficiency.

EXHIBIT 2. EFFICIENCY OF PORTFOLIO PROTECTION STRATEGIES



Each graph shows the scatterplot of maximum loss and maximum underperformance of corresponding portfolio protection strategy based on numerical simulations. Each point represents a single simulated episode where maximum loss and underperformance are computed over all intervals of a fixed length within the episode (in grey). Each graph also contains a scatterplot corresponding to the approximation of the optimal portfolio protection described in the academic paper (in black).
Source: LOIM. For illustrative purposes only

4 We assumed that positive and negative returns occur with the same probability. This assumption is needed only to simulate possible market scenarios. We do not compute probabilities of different outcomes in our analysis.

5 OBPI was implemented through a number of overlapping strategies replicating an at-the-money call option where one strategy is rolled over every week. We effectively assume that the investor is equally averse to loss and missing-out.

6 For example, OBPI would yield the same shape if we plotted its outcomes with the loss and underperformance measured over the whole investment horizon. Indeed, independent of the index path, the investment ends up either with a loss equal to the option price or an underperformance of the same magnitude.

7 The index weight was constrained to not exceed 1 to avoid leverage. The multiplier and the maximum loss of CPPI were calibrated to minimise the average of the maximum of the two outcomes over all simulations. Taking the maximum means that we treat loss and missing-out in a symmetric way.

In contrast to CPPI, OBPI yields a noticeably more symmetrical distribution since call options protect both absolute and relative performance. While this solution is less prone to extreme underperformance (the vertical spread), it gives up on the downside protection resulting in a wider spread of maximum losses breaching the level of 0.2. Overall, both CPPI and OBPI seem to be substantially inferior to the benchmark.

Balanced strategy clearly provides the most efficient solution among the three as evidenced by a substantially narrower spread of its outcomes. While it is still not as good as the benchmark, we should keep in mind that our comparison is made in a theoretical environment most favourable for model-based solutions.

Let us be clear, the superior behaviour of balanced strategy is not a result of our lucky guess of the magic formula. This time, ingredients are more important than recipe. In our view, an efficient portfolio protection strategy must ensure a balance between the downside protection with upside participation.[8] Here we chose the simplest formula that does it explicitly by incorporating right ingredients – portfolio loss and underperformance over a rolling window.

However, we can imagine many other analytical forms that can achieve a similar effect using the same inputs. It is worth noting that while OBPI has all the right ingredients for a fixed investment horizon, it cannot be efficiently adopted to the objective of portfolio protection over a rolling window, therefore losing its theoretical advantage.

Conclusion

Protection of a multi-client portfolio is challenging since such mechanisms should insure the portfolio performance simultaneously over multiple intervals. As a consequence, the optimal portfolio protection represents a complex mechanism that lacks interpretability. In this note we argued that a simple solution that seeks to balance portfolio loss and its upside participation can offer a good alternative, being far superior to the classical solutions such as Option Based Portfolio Insurance and Constant Proportionate Portfolio Insurance. This balanced strategy is rather straightforward to implement, and it yields decent results in a simulation environment.

[8] In our previous paper on the drawdown control (Drawdown control solutions: beware opportunity costs | Lombard Odier) we raised a similar point suggesting a modification to CPPI by incorporating the portfolio underperformance relative to the index. Such a modified strategy, of course, can no more guarantee the cap on the portfolio loss.

IMPORTANT INFORMATION